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Engineering Notes

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Feedback Control for Spacecraft Rendezvous and Docking

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Introduction

RENDEZVOUS and docking maneuvers between an active spacecraft and a passive target will be of increasing importance as space station applications emerge. A popular approach to the terminal guidance problem has been to utilize the linearized equations of motion relative to the target orbit.[1] Several researchers, e.g., Refs. 2–4, have investigated optimal minimum-fuel rendezvous maneuvers. In addition, Tapley and Fowler[5] and Yu[6] present terminal guidance schemes for rendezvous.

Many of the previous terminal rendezvous problems have been solved without constraints on the approach direction between the two spacecraft. For a realistic spacecraft docking maneuver, the approach direction is constrained along the target docking axis. Guelman and Aleshin[4] obtained minimum-fuel rendezvous maneuvers with a fixed approach direction by dividing the trajectory into two stages. The result is an optimal open-loop control program that is obtained by solving the corresponding two-point boundary value problem. Lopez and McInnes[7] utilized a Lyapunov approach to design a guidance scheme that ensures obstruction avoidance and completes the rendezvous along a desired direction. The resulting guidance scheme maintains a monotonically decreasing Lyapunov function by commanding impulsive velocity increments.

In this Note, a feedback control scheme is developed for terminal planar rendezvous along a prescribed docking axis. Continuous thrust acceleration is controlled along two orthogonal axes. The control law guarantees rendezvous along a desired approach direction with a desired approach speed relative to the target. Numerical simulation of a rendezvous and docking maneuver is presented.

Rendezvous Control

Equations of Motion for Control Law Development

The planar rendezvous problem consists of controlling the active spacecraft so that it docks with a passive target satellite in a circular orbit along a prescribed docking axis. For development of the feedback control scheme, motion of the active spacecraft relative to the target is governed by the linear Clohessy–Wiltshire (CW) equations[1]:

$$\ddot{x} = 3\omega^2 x + 2\omega\dot{y} + a_x \tag{1}$$

$$\ddot{y} = -2\omega\dot{x} + a_y \tag{2}$$

The rotating polar xy frame is fixed to the target spacecraft, and its origin moves in a circular orbit at a constant angular rate ω. The $+x$ axis is along the radial direction from Earth to the target, and the $+y$ axis is the downtrack distance along the circular target orbit. For this analysis, $x = \delta r = r - r^*$ and $y = r^*\delta\lambda$, where r^* is the radius of the circular target orbit and $\delta\lambda$ is the longitude angle error between the active and target spacecraft. This choice of coordinates expands the linear range of the CW equations to a torus about the circular target orbit, and therefore only the radial coordinate x must be small compared to r^* for a good linear approximation of the governing dynamics. Thrust acceleration components of the active spacecraft are designated by a_x and a_y.

Control Laws

The objective is to complete the rendezvous maneuver along an arbitrary docking direction. Docking direction can be defined by the angle θ measured from the $+x$ (radial) axis, i.e., $\theta = \tan^{-1}(y/x)$. Therefore, it is advantageous to express the equations of motion in a rotating frame fixed to the line-of-sight (LOS) direction to the target. The LOS and transverse (normal to the LOS) velocity components of the active spacecraft are

$$v_\rho = \dot{x}\cos\theta + \dot{y}\sin\theta \tag{3}$$

$$v_\theta = -\dot{x}\sin\theta + \dot{y}\cos\theta \tag{4}$$

The LOS and transverse acceleration components of the active spacecraft are obtained by taking the time derivatives of Eqs. (3) and (4):

$$\dot{v}_\rho = \ddot{x}\cos\theta - \dot{x}\dot{\theta}\sin\theta + \ddot{y}\sin\theta + \dot{y}\dot{\theta}\cos\theta \tag{5}$$

$$\dot{v}_\theta = -\ddot{x}\sin\theta - \dot{x}\dot{\theta}\cos\theta + \ddot{y}\cos\theta - \dot{y}\dot{\theta}\sin\theta \tag{6}$$

The CW equations (1) and (2) are substituted for $\ddot{x}$ and $\ddot{y}$ in Eqs. (5) and (6). The LOS thrust acceleration $a_\rho = a_x\cos\theta + a_y\sin\theta$ and the transverse thrust acceleration $a_\theta = -a_x\sin\theta + a_y\cos\theta$ also are substituted to give

$$\dot{v}_\rho = [3\omega^2 x + \dot{y}(2\omega + \dot{\theta})]\cos\theta - \dot{x}(2\omega + \dot{\theta})\sin\theta + a_\rho \tag{7}$$

$$\dot{v}_\theta = -[3\omega^2 x + \dot{y}(2\omega + \dot{\theta})]\sin\theta - \dot{x}(2\omega + \dot{\theta})\cos\theta + a_\theta \tag{8}$$

Because the goal is to control the docking angle θ, it is advantageous to express Eq. (8) as a second-order differential equation in θ. Differentiating $v_\theta = \rho\dot{\theta}$, where $\rho = \sqrt{(x^2 + y^2)}$ is the LOS range, with respect to time results in

$$\dot{v}_\theta = \dot{\rho}\dot{\theta} + \rho\ddot{\theta} \tag{9}$$

Therefore, solving for the angular acceleration gives

$$\ddot{\theta} = (1/\rho)\left\{-[3\omega^2 x + \dot{y}(2\omega + \dot{\theta})]\sin\theta - \dot{x}(2\omega + \dot{\theta})\cos\theta - v_\rho\dot{\theta} + a_\theta\right\} \tag{10}$$

The LOS and transverse thrust acceleration controls a_ρ and a_θ are generated by using a feedback linearization approach. The two respective controls have the forms

$$a_\rho = u_\rho + w_\rho, \qquad a_\theta = u_\theta + w_\theta \tag{11}$$

where the components w_ρ and w_θ are the nonlinear controls required to linearize the equations of motion in the LOS frame:

$$w_\rho = -[3\omega^2 x + \dot{y}(2\omega + \dot{\theta})]\cos\theta + \dot{x}(2\omega + \dot{\theta})\sin\theta \tag{12}$$

$$w_\theta = [3\omega^2 x + \dot{y}(2\omega + \dot{\theta})]\sin\theta + \dot{x}(2\omega + \dot{\theta})\cos\theta + v_\rho\dot{\theta} \tag{13}$$

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Using the preceding controls (11–13), the equations of motion in the LOS frame become

$$\dot{v}_\rho = \ddot{\rho} = u_\rho, \qquad \ddot{\theta} = u_\theta/\rho \tag{14}$$

For the rendezvous and docking problem, the objective is to drive the angle θ to a desired docking direction and acquire the proper docking axis at a sufficient range from the target. Therefore, the following position errors are defined:

$$\rho_e = \rho - \rho^*, \qquad \theta_e = \theta - \theta^* \tag{15}$$

where ρ^* and θ^* are fixed values that define the docking axis corridor. Because $\ddot{\rho}_e = \ddot{\rho}$ and $\ddot{\theta}_e = \ddot{\theta}$, the error dynamics become

$$\ddot{\rho}_e = u_\rho, \qquad \ddot{\theta}_e = u_\theta/\rho \tag{16}$$

A simple proportional–derivative controller is proposed:

$$u_\rho = -k_1\dot{\rho}_e - k_2\rho_e \tag{17}$$

$$u_\theta = -\rho(k_3\dot{\theta}_e + k_4\theta_e) \tag{18}$$

where the four gains k_1, k_2, k_3, and k_4 are all positive. When control laws (12), (13), (17), and (18) are used, the equations of motion in the LOS frame are transformed into two uncoupled, homogeneous, linear second-order differential equations. Therefore, convergence to the desired docking axis is guaranteed (with $k_i > 0$) and the desired transient response characteristics can be obtained by tuning the feedback gains.

Numerical Results

Numerical simulation of a rendezvous and docking maneuver using the proposed control scheme is presented. The target satellite is in a circular low Earth orbit with an altitude of 407 km. Motion of the active spacecraft is determined by numerically integrating the governing nonlinear system (inverse-square gravitational field) of differential equations. Because the CW coordinates are required in the control laws, the polar coordinates $(x, y, \dot{x}, \dot{y})$ are computed by differencing the position and velocity of the active and target spacecraft. The LOS and transverse thrust acceleration controls a_ρ and a_θ are transformed to the respective components in the polar frame. The nonlinear equations of motion are made nondimensional by defining reference length and time units that correspond to the circular target orbit so that the period of the target orbit is 2π and the angular rate is unity. Numerical integration of the nondimensional nonlinear differential equations is performed by a variable-order Runge–Kutta routine with a local error tolerance of 10^{-12} for step-size control.

The feedback gains were selected by specifying a settling time around one orbital period and a damping ratio of about 0.9. In nondimensional units, the gains used here are $k_1 = 0.7$, $k_2 = 0.1$, $k_3 = 2.6$, and $k_4 = 2.1$. Note that the transverse thrust acceleration control law (18) also is multiplied by LOS range ρ.

It is desired that the active spacecraft reach the docking axis at a range of at least 300 m and approach the target along the fixed direction at a LOS rate of 0.2 m/s. Therefore, the rate feedback term in the LOS thrust acceleration control law (17) utilizes the range-rate error

$$u_\rho = -k_1\left(v_\rho - v_\rho^*\right) - k_2(\rho - \rho^*) \tag{19}$$

where $\rho^* = 300$ m and $v_\rho^* = -0.2$ m/s. Once the chaser begins the approach along the docking corridor and is sufficiently close to the target ($\rho < 300$ m), the range error feedback term $\rho - \rho^*$ is set to zero.

The active spacecraft is initially 1000 m below and 1000 m downtrack of the target. It is assumed that the active spacecraft is in a nearly circular orbit and therefore the initial velocity is $\dot{x}(0) = -0.1$ m/s and $\dot{y}(0) = +1.69$ m/s relative to the target. These initial conditions will cause the active spacecraft to drift away from the target in the absence of propulsive controls. The desired docking angle is $\theta^* = -30$ deg, which is in the opposite quadrant as the initial position of the chaser.

Figure 1 shows the rendezvous and docking trajectory in the xy plane and Fig. 2 shows the time histories of the LOS range ρ and docking angle θ. The chaser reaches the target in 151.9 min and docks along the desired direction with a closing velocity of 0.22 m/s. Note that the desired docking axis is achieved with no overshoot in about 60 min at a range of about 1770 m. After 60 min, the chaser essentially moves along the desired LOS with a range rate that approaches -0.2 m/s. The final docking angle θ is -29.9997 deg. Figure 3 shows the thrust acceleration components a_ρ and a_θ. The total velocity increment (ΔV) required for this maneuver is 52.2 m/s and is computed by integrating the magnitude of the total thrust acceleration.

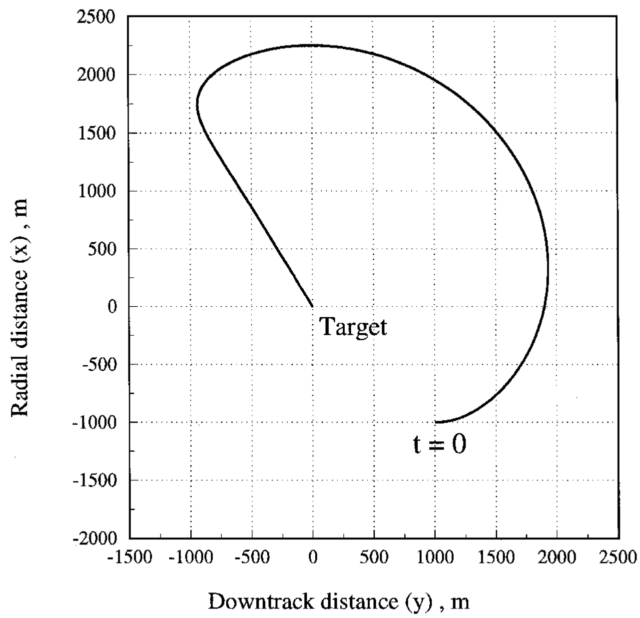

Fig. 1 Rendezvous and docking maneuver.

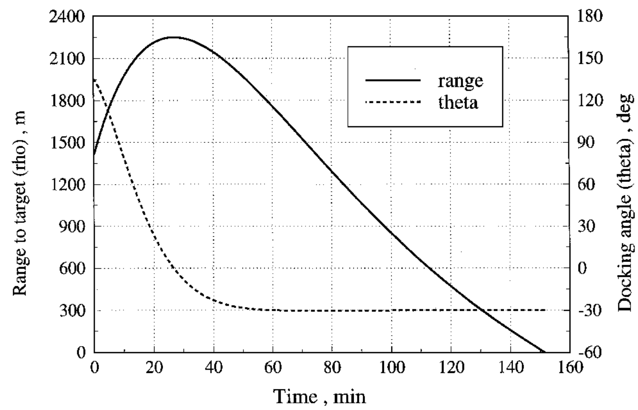

Fig. 2 Range and docking angle for docking maneuver.

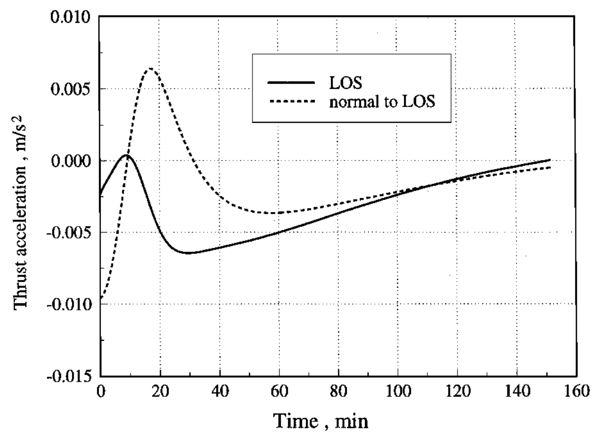

Fig. 3 Thrust acceleration for docking maneuver.

Conclusions

A feedback control law for performing planar rendezvous and docking maneuvers is presented. The control law uses a feedback linearization approach and guarantees tracking along an arbitrary fixed docking direction with a desired approach speed. Although the control scheme is based on the linear CW equations, a simulation of the nonlinear system shows that the control law results in accurate rendezvous and docking conditions. The feedback control law is relatively simple and therefore may be a potential onboard guidance scheme for autonomous docking maneuvers.

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Uncertainty Modeling in Aerospace Flexible Structures

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Introduction

THIS Note describes an alternative modeling scheme for the uncertainty present in aerospace flexible structures by a nonconservative characterization of the regions containing the perturbed eigenvalues of the dynamical system. In general, the term *flexible structure* is commonly used for linear systems with oscillatory properties characterized by a strong amplification of harmonic signals at the natural frequencies, and its transfer function poles are complex conjugate, typically with a small real part.[1] For the case of aerospace applications, these structures will have many resonant low frequencies with damping properties of approximately 0.5% critical, and often they appear in closely spaced clumps throughout the control system bandwidth. Frequently, the natural frequencies and damping factors (ω_i and ζ_i) of the structural modes that are included in the model are not known exactly.

Such inaccuracies or errors can be represented in two different ways. In the frequency domain three common ways of describing unstructured uncertainty are used. The difference between the nominal model and the real plant can be presented as additive or multiplicative uncertainty. A constraint is that these families of models must have the same number of right-half plane poles as the nominal one. Another description[2] is based on the uncertainty on the coprime factors. These descriptions do not have the previous limitation but may lead to conservative representations for lightly damped structures.[3] In the time domain uncertainty modeling can be described by the model parameters. In particular, the uncertainty can be expressed by variations in the A, B, C, or D matrices of the state-space representation of the model, i.e., $G(s) \overset{s}{=} (A, B, C, D)$.

The approach adopted here is motivated by that of Smith,[4] which describes the uncertain model through a linear fractional transformation (LFT), where the uncertainty present in the damping factor and natural frequency of each flexible mode is modeled through perturbations that affect the eigenvalues of the real modal dynamic matrix of the model. Hence, a nonconservative uncertainty description of these type of structures can be used to design robust controllers by standard methods such as H_∞ or μ-synthesis.[5] The object of this work is to point out that the real block diagonal perturbations (highly structured uncertainty) derived from the natural frequencies and damping factors can be described with no conservativeness as unstructured uncertainty. Therefore, optimal controllers may be obtained for this class of uncertain models without having to take into consideration the real nature of the uncertainty and its particular structure. To illustrate this uncertainty modeling technique, the results are applied to the synthesis of controllers for a flexible structure that is well-known in the literature.[6] In this case they were designed to increase the damping in the lightly damped low-frequency modes without affecting its neglected higher-frequency dynamics.

Uncertainty Modeling and Perturbed Eigenvalue Regions

Let $A \in \mathbb{R}^{2n\times 2n}$ be a modal matrix with n pairs of complex conjugate nominal eigenvalues $\lambda_{i\pm} = \alpha_i \pm j\beta_i$ and $W \in \mathbb{R}^{2n\times 2n}$ a diagonal weighting matrix. Now consider the uncertainty as perturbations to the nominal system eigenvalues represented by a particular LFT that replaces A by $A + W\Delta$, where $\Delta = \mathrm{diag}(\Delta_i)$, $\Delta_i \in \mathbb{R}^{2\times 2}$, $\bar{\sigma}(\Delta_i) \le 1$, $i = 1, \ldots, n$, and $\bar{\sigma}(\cdot)$ denotes the maximum singular value. For the specific LFT to be used, the A matrix must be in real modal form:

$$A = \mathrm{diag}(A_1, \ldots, A_n), \qquad A_i = \begin{bmatrix} \alpha_i & -\beta_i \\ \beta_i & \alpha_i \end{bmatrix} \tag{1}$$

$$W = \mathrm{diag}(w_1 I_2, \ldots, w_n I_2), \qquad w_i > 0 \tag{2}$$

The matrix W defines $2n$ disks in the complex plane centered at the eigenvalues of A, where w_i represents the disks radii located at $\lambda_{i\pm}$. Now, consider the problem of describing the uncertain model with inputs u and outputs y via an upper LFT formulation $F_u[Q(s), \Delta]$ so that all uncertainty can be represented as

$$y = \{C[sI - (A + W\Delta)]^{-1}B + D\}u \tag{3}$$

where

$$F_u[Q(s), \Delta] = Q_{21}\Delta(I - Q_{11}\Delta)^{-1}Q_{12} + Q_{22} \tag{4}$$

and the state-space realization of $Q(s)$ can be written as

$$Q(s) \overset{s}{=} \left[\begin{array}{c|cc} A & W & B \\ \hline I & 0 & 0 \\ C & 0 & D \end{array}\right] \tag{5}$$

As can be seen from Eq. (3), $F_u[Q(s), \Delta]$ clearly allows the inclusion of perturbations to the matrix A, mapping it to $A + W\Delta$. The contribution of this work is an extension of Theorem 4 from Ref. 4 to structured real block perturbations that appear in modal realizations of large space structures (LSSs); it provides a nonconservative description of the preceding uncertain model by means of unstructured uncertainty. This extension is implicit in the proof of the theorem mentioned before, although it is not explicitly stated. Next, we show this result.

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